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$$E(X) = \sum_{i=1}^4 x_i p_i = 0.1 \times 7 + 8 \times 0.3 + 9 \times 0.3 + 10 \times 0.3 = 8.8$$

$$E(Y) = \sum_{i=1}^4 y_i p_i = 0.2 \times 7 + 8 \times 0.3 + 9 \times 0.5 + 10 \times 0.1 = 8.5$$

2.?????????????

4.1.2 4.1.1 $E(X)$???????????????????? $f(x)$,?

$$E(X) = \int_{-\infty}^{+\infty} xf(x)dx$$

4.1.4 ???????X?????????

$$F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x}{4}, & 0 < x \leq 4 \\ 1, & x > 4 \end{cases}$$

$E(X)$?

$F(x)$?????????

$$f(x) = \begin{cases} 0, & x \leq 0 \\ \frac{1}{4}, & 0 < x \leq 4 \\ 0, & x > 4 \end{cases}$$

$$E(x) = \int_{-\infty}^{+\infty} xf(x)dx = \int_0^4 \frac{1}{4}x dx = \frac{1}{4} \frac{x^2}{2} \Big|_0^4 = 2$$

4.1.6 ???????X?????????

$$f(x) = \begin{cases} ax+b, & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

$$E(x) = \frac{7}{12}, \quad a, b \text{????????????????}$$

?: ??????????????

$$E(X) = \int_0^1 x(ax+b)dx = \left(\frac{1}{3}ax^3 + \frac{1}{2}bx^2 \right) \Big|_0^1 = \frac{a}{3} + \frac{b}{2} = \frac{7}{12}$$

$$\int_{-\infty}^{+\infty} (ax+b) dx = \int_0^1 (ax+b) dx = \left(\frac{1}{2}ax^2+bx\right)\Big|_0^1 = \frac{a}{2} + b = 1$$

?? a=1, b=1/2,

??? $0 \leq x \leq 1$??

$$F(x) = \int_0^x f(t) dt = \int_0^x (t + \frac{1}{2}) dt = \left(\frac{t^2}{2} + \frac{t}{2}\right)\Big|_0^x = \frac{x^2 + x}{2}$$

??????????

$$F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x^2 + x}{2}, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

4.1.2 ????????????

X ????? $g(x)$ $Y=g(x)$ $E[g(x)]$,
 ????? $E[g(x)]$,
 ?????

4.1.1 X $Y=g(x)$, $E(Y)$???

(1). X ????????**

$$P\{X=x_i\} = p_i, i=1,2,3,\dots$$

$$E(Y)=E[g(x)]=\sum_{i=1}^{\infty}g(x_i)p_i$$

(2.) X ???????? $f(x)$ $\int_{-\infty}^{\infty}g(x)f(x)dx$?????
 ?????

$$E(Y)=E[g(x)] = \int_{-\infty}^{\infty}g(x)f(x)dx$$

4.1.6 ????? X ?????

$x=x_i$	-2	0
P	0.3	0.1

$$E(2X+3), E(X^2-1)$$

$$E(2X+3) = 2 \times (-2) \times 0.3 + 2 \times 0 \times 0.1 = -1.2$$

$$E(X^2-1)=4 \times 0.3 + 0^2 \times 0.1 = 1.2$$

1????4.1.1???? $E[g(X)]$ $g(X)$ X
 ?????

????4.1.1 ?????.

4.1.3 ?????

$$E(C) = C$$

???? c ????, $X \equiv C$????

$$P\{X=c\} = 1 \quad E(c) = cP\{X=c\} = c$$

(2) $E(CX) = CE(X)$

$P\{X=x_i\}=p_i, i=1,2,3,\dots$

$E(CX) = \sum_{i=1}^{\infty} (Cx_i)p_i = C \sum_{i=1}^{\infty} x_i p_i = CE(x)$

$f(x)$, $E(CX) = \int_{-\infty}^{\infty} Cxf(x)dx = C \int_{-\infty}^{\infty} xf(x)dx = CE(x)$

(3) $E(X+Y) = E(X) + E(Y)$

(4) $E(\sum_{i=1}^n X_i) = \sum_{i=1}^n E(X_i)$

(5) $E(XY) = E(X)E(Y)$

$$E(X^2) = E[X(X-1)+X] = E[X(X-1)] + E(X) = \lambda^2 + \lambda$$

$$D(X) = E(X^2) - [E(X)]^2 = \lambda$$

4.2.6 $X \sim U(a,b)$ $E(X), D(X)$

$f(x)$

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$E(X) = \frac{1}{b-a} \int_a^b x dx = \frac{a+b}{2}$$

$$D(X) = E(X^2) - [E(X)]^2 = \frac{1}{b-a} \int_a^b x^2 dx - \left(\frac{a+b}{2}\right)^2 = \frac{(b-a)^2}{12}$$

4.2.7 $X \sim E(\lambda)$ $E(X), D(X)$

$f(x)$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$E(X) = \int_0^{\infty} x \lambda e^{-\lambda x} dx = \frac{1}{\lambda}$$

$$E(X^2) = \frac{2}{\lambda^2}$$

$$D(X) = \frac{1}{\lambda^2}$$

4.2.8 $X \sim N(\mu, \sigma^2)$ $E(X), D(X)$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$E(X) = \mu, E(X^2) = \sigma^2 + \mu^2$$

$$D(X) = \sigma^2$$

?? 4.2.2 ?????? X ?????? $E(X)=\mu$, $D(X)= \sigma^2$ \$,
?????? ϵ \$, ???

$$P\{|X- \mu | \geq \epsilon\} \leq \frac{\sigma^2}{\epsilon^2} \quad ?$$

$$P\{|X- \mu | < \epsilon\} \geq 1- \frac{\sigma^2}{\epsilon^2} \quad ?$$

???????

?\$cov(X,Y), D(X+Y)\$

??\$cov(X,Y) = E(XY)- E(X)E(Y)\$

$$f_X(x) = \int_{x^0}^1 f_X(x,y)dy = 8x \int_{x^0}^1 ydy = 4xy^2 \Big|_x^1 = 4x(-x^2+1)$$

$$f_Y(y) = \int_{y^0}^1 f_Y(x,y)dx = 8y \int_0^y xdx = 4yx^2 \Big|_y^0 = 4y^3$$

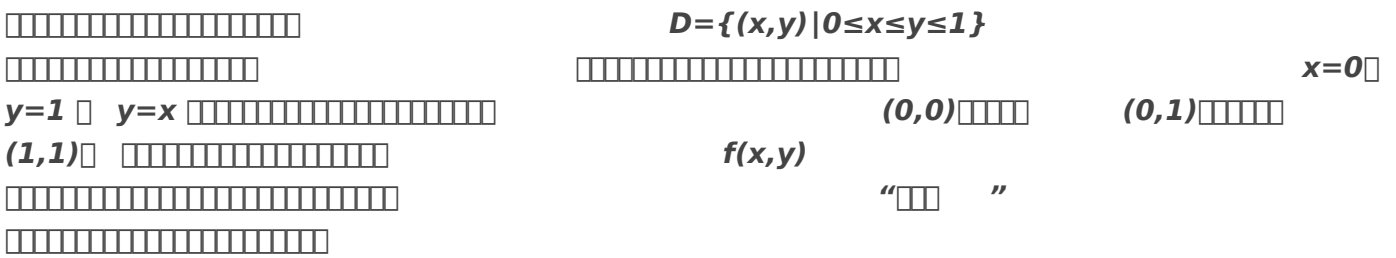
\$\$ f_X(x)=\begin{cases} 4x(-x^2+1), & \text{ } 0 \leq x \leq 1 \\ 0, & \text{ } \end{cases} \\ f_Y(y)=\begin{cases} 4y^3, & \text{ } 0 \leq y \leq 1 \\ 0, & \text{ } \end{cases} \$\$

$$E(X) = \int_{-\infty}^{+\infty} xf_X(x)dx = 8/15$$

$$E(Y) = \int_{-\infty}^{+\infty} yf_Y(y)dy = 4/5$$

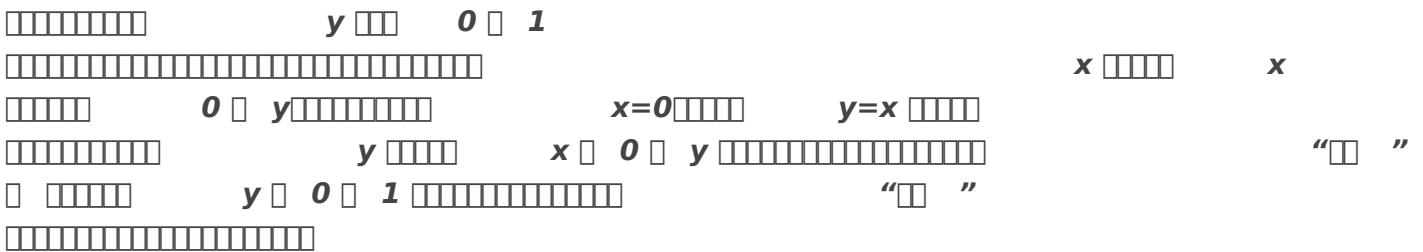
$$E(XY) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} xyf(x,y)dxdy$$

$$= \int_0^1 dx \int_{x^0}^1 (xy \times 8xy)dy = 4/9$$

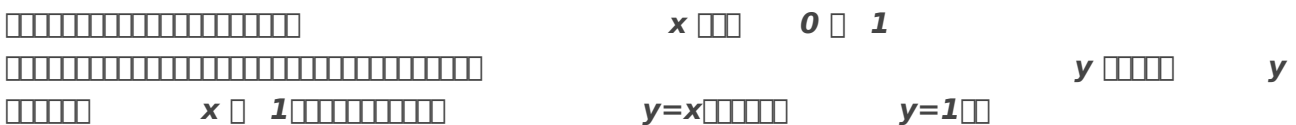


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?? x ??????? y ??????



?? y ??????? x ??????



$\int_0^1 \int_0^1 xy \, dx \, dy = \int_0^1 \left[\frac{1}{2}x^2 y \right]_0^1 dy = \int_0^1 \frac{1}{2}y \, dy = \frac{1}{4}$

“ ”

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y) = \frac{4}{9} - \frac{8}{15} \times \frac{4}{5} = \frac{4}{225}$$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f_X(x) dx = 1/3$$

$$E(Y^2) = \int_{-\infty}^{+\infty} y^2 f_Y(y) dy = 2/3$$

$$D(X+Y) = D(X) + D(Y) + 2\text{Cov}(X, Y)$$

$$= E(X^2) - [E(X)]^2 + E(Y^2) - [E(Y)]^2 + \frac{4}{225} = 1/9$$

4.3.3 $\rho = \frac{\text{Cov}(X, Y)}{\sqrt{D(X)D(Y)}}$ $D(X) > 0, D(Y) > 0, \rho \in [-1, 1]$
 $\rho = 0$ X Y

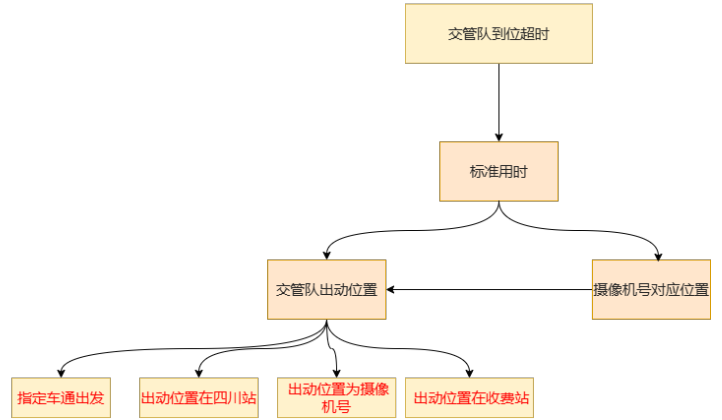
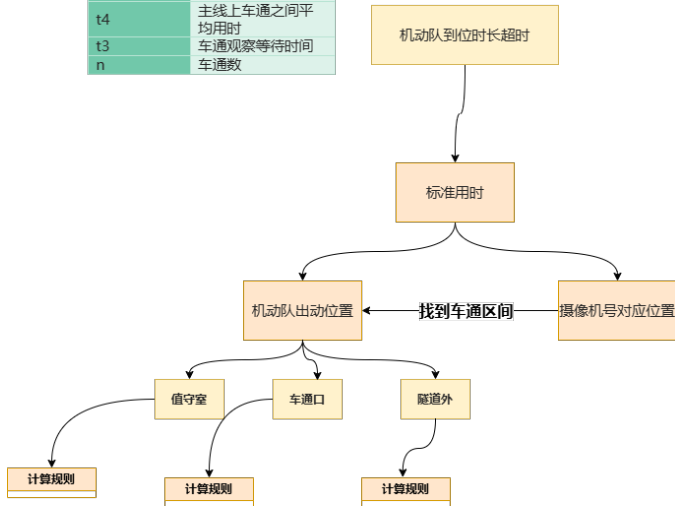
4.3.4

- $|\rho| \leq 1$
 - $\rho = 0$
 - $D(X) > 0, D(Y) > 0$
- $|\rho| = 1$ $a, b (a \neq 0)$ $P\{Y = aX + b\} = 1$,
 $a > 0$ $\rho = 1, a < 0$ $\rho = -1$.



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字母名称	说明
t	机动队到位标准用时
t1	准备时间 单位秒
t2	洞内车通之间平均用时
t4	主线上车通之间平均用时
t3	车通观察等待时间
n	车通航



比如左四车通
 $n=5-4$
 $(1/2+1) \times 2$
 八车通
 $n=8-6-2$
 $(1/2+2) \times 2$

·值守室出发: (位置在5#-6#之间)
 值守室到达最近车通时间 $(1/2+n) \times t_2$

???

?????



$\theta_k = A_k$ \end{cases} \$\$

?????? $\hat{\theta}_1$, $\hat{\theta}_2$, ??????? $\hat{\theta}_1$,
 $\hat{\theta}_2$, , $\hat{\theta}_k$, ??? $\theta_l = (X_1, X_2, \dots, X_n)$???
 θ_l ?????, ??????? $\hat{\theta}_l = \hat{\theta}_l(x_1, x_2, \dots, x_n)$
????????, ?????????????????**



7.1.2 ??????

?7.1.1

???X???????

$$f(x) = \begin{cases} (\alpha + 1)x^\alpha, & 0 \leq x < 1, \alpha > -1, \\ 0, & \text{otherwise} \end{cases}$$

Let X_1, X_2, \dots, X_n be independent random variables with the same distribution as X .

??

$$\overline{X} = \mu = E(x) = \frac{1}{n} \sum_{i=1}^n X_i$$

$$\begin{aligned} \mu = E(x) &= \int_{-\infty}^{+\infty} xf(x)dx = \int_0^1 x(\alpha + 1)x^\alpha dx \\ &= \int_0^1 (\alpha + 1)x^{\alpha+1}dx = \int_0^1 (\alpha + 1)x^{\alpha+1} + x^{\alpha+1}dx \\ &= \left(\frac{\alpha + 1}{\alpha + 2} x^{\alpha+2} + \frac{1}{\alpha + 2} x^{\alpha+2} \right) \Big|_0^1 = \frac{\alpha + 1}{\alpha + 2} \end{aligned}$$

??:

$$\overline{X} = \frac{\alpha + 1}{\alpha + 2}$$

$$\alpha = \frac{1 - 2\overline{X}}{\overline{X} - 1}$$

7.1.2 Let $X_1, X_2, X_3, \dots, X_n$ be independent random variables with the same distribution as X .

$$\mu_1 = E(x) = A_1, \mu_2 = E(x^2) = A_2$$

$$E(X) = \frac{a+b}{2}$$

$$E(x^2) = \frac{1}{b-a} \int_a^b x^2 dx = \frac{(b-a)^2}{12} + \frac{(a+b)^2}{4}$$

?

$$\begin{cases} a+b = 2\mu, \\ b-a = \sqrt{12(\mu_2 - \mu_1^2)} \end{cases}$$

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$$a = \mu_1 - \sqrt{3(\mu_2 - \mu_1^2)}, \quad b = \mu_1 + \sqrt{3(\mu_2 - \mu_1^2)}$$

$$A_1 = \frac{1}{n} \sum_{i=1}^n X^1_i, \quad A_2 = \frac{1}{n} \sum_{i=1}^n X^2_i,$$

$$\left(\frac{1}{n} \sum_{i=1}^n X^2_i - \overline{X^2} \right) = \frac{1}{n} \sum_{i=1}^n (X_i - \overline{X})^2$$

??a?b??????

$$a = A_1 - \sqrt{3(A_2 - A_1^2)} = \overline{X} - \sqrt{3 \left(\frac{1}{n} \sum_{i=1}^n (X_i - \overline{X})^2 \right)} = \overline{X} - \sqrt{3 \frac{n-1}{n} S^2} \\ b = A_1 + \sqrt{3(A_2 - A_1^2)} = \overline{X} + \sqrt{3 \left(\frac{1}{n} \sum_{i=1}^n (X_i - \overline{X})^2 \right)} = \overline{X} + \sqrt{3 \frac{n-1}{n} S^2}$$

??? ???????



6.2

6.2.1 $(X_1, X_2, \dots, X_n) \sim N(\mu, \sigma^2)$ $E(\overline{X}) = \mu$, $D(\overline{X}) = \frac{\sigma^2}{n}$

6.2.2

$(X_1, X_2, \dots, X_n) \sim N(\mu, \sigma^2)$

$X_i \sim N(\mu, \sigma^2), (i=1, 2, \dots, n)$ $E(\overline{X}) = \mu$, $D(\overline{X}) = \frac{\sigma^2}{n}$

$D(\overline{X}) = \frac{\sigma^2}{n}$, $D(X_i) = \sigma^2$

$\overline{X} \sim N(\mu, \frac{\sigma^2}{n})$, $Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$

$E(S^2) = \sigma^2$

6.2.3

1. χ^2

6.2.2 $(X_1, X_2, \dots, X_n) \sim N(0, 1)$ $\chi^2 = \sum_{i=1}^n X_i^2$

χ^2

(1). $\chi_1^2 \sim \chi^2(n_1), \chi_2^2 \sim \chi^2(n_2)$, $\chi_1^2 + \chi_2^2 \sim \chi^2(n_1 + n_2)$

(2). $E(\chi^2) = n, D(\chi^2) = 2n$

$(X_1, X_2, \dots, X_n) \sim N(\mu, \sigma^2)$ $\chi^2 = \sum_{i=1}^n \frac{(X_i - \mu)^2}{\sigma^2} \sim \chi^2(n)$

6.2.1 $(X_1, X_2, \dots, X_n) \sim N(\mu, \sigma^2)$ $\overline{X} \sim N(\mu, \frac{\sigma^2}{n})$ $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

6.2.3 $P(X > c) = \alpha$, $0 < \alpha < 1$, $c = c(\alpha)$
 $P(X > c) = \alpha$

2. t

6.2.2 $X \sim N(0, 1)$, $Y \sim \chi^2(n)$, $T = \frac{X}{\sqrt{\frac{Y}{n}}}$, $T \sim t(n)$

6.2.2 (X_1, X_2, \dots, X_n) , $X \sim N(\mu, \sigma^2)$, $T = \frac{\overline{X} - \mu}{S/\sqrt{n}}$, $T \sim t(n-1)$

2. F

6.2.3 $X \sim \chi^2(m)$, $Y \sim \chi^2(n)$, $F = \frac{X/m}{Y/n}$, $F \sim F(m, n)$

$T \sim t(n)$, $T^2 = \frac{X^2/1^2}{[\sqrt{\frac{Y}{n}}]^2} \sim F(1, n)$
 F

(1). $F \sim F(m, n)$, $\frac{1}{F} \sim F(n, m)$

(2). $F_{\alpha}(n, m) = \frac{1}{F_{1-\alpha}(m, n)}$

6.2.4 (X_1, X_2, \dots, X_n) , $X \sim N(\mu_1, \sigma_1^2)$, (Y_1, Y_2, \dots, Y_n) , $Y \sim N(\mu_2, \sigma_2^2)$, (X, Y) , S_1^2, S_2^2 , $F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(m-1, n-1)$

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\$\$ □□□ = \begin{cases} \sigma^2 □□ & Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \\ Z_{\frac{\alpha}{2}} \sigma & Z = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}, □□ (n-1) \\ t_{\frac{\alpha}{2}} \end{cases} \$\$