??? ?????????

- 4.1
- 4.1.2
- 4.2
- 4.1.3

4.1 ?????????

[TOC]

????

??????????????????

$x=x_i$	0	1	2	3	...	M	□□
□□□□	m_0	m_1	m_2	m_3	...	m_M	N
□□□□	p^*_0	p^*_1	p^*_2	p^*_3	...	p^*_4	1

?? $N=\Sigma_{i=1}^{M}m_i, p^*_i = m_i /N $

????????????????

$$\overline{X} = \Sigma_{i=1}^{M} p^*_ix_i $$

???????????????????????????????????????
$\Sigma_{i=1}^{M} p_ix_i $???????????????????????????????????

1. ????????????

???4.1.1 ?X????????? ??????$$P(X=x_i) = p_i, i=1,2,3,...$$

?$\Sigma_{i=1}^{+\infty} p_ix_i$????????X??????????$$E(X) = \Sigma_{i=1}^{\infty} p_ix_i$$

?$\Sigma_{i=1}^{+\infty} p_i|x_i|$??????$X$????????

?4.1.1 ?????????X?Y???????????????X,Y??????

$X=x_i$	7	8	9	10
P	0.1	0.3	0.3	0.3

$Y=y_i$	7	8	9	10
P	02	0.3	0.5	0.1

???????

? $E(X) = \Sigma_{1}^{4}x_ip_i = 0.1\times7 + 8\times0.3 + 9\times0.3 + 10\times0.3 = 8.8$

$E(Y) = \Sigma_{1}^{4}y_ip_i = 0.2\times7 + 8\times0.3 + 9\times0.5 + 10\times0.1 = 8.5$

2.????????????

?? 4.1.2 4.1.1 ?X????????? ?????$f(x)$,?

$$\int_{-\infty}^{+\infty}xf(x)dx$$????????X??????$E(X) = \int_{-\infty}^{+\infty}xf(x)dx$

? 4.1.4 ??????X??????

$$ F(x) = \begin{cases} 0, \text{ } x \leq 0 \\ \frac{x}{4}, \text { } 0 \lt x \leq 4 \\ 1, x \gt 4 \end{cases} $$

? $E(X)$?

??F(x)??????

$$ f(x) = \begin{cases} 0, x \leq 0□ x \gt 4, \\ \frac{1}{4}, 0 \lt x \leq 4 \end{cases} $$

??$E(x) = \int_{-\infty}^{+\infty}xf(x)dx = \int_{0}^{4}\frac{1}{4}xdx = \frac{1}{4}\frac{x^2}{2}\big|^4_0 = 2$

? 4.1.6 ?????X??????

$$ f(x) = \begin{cases} ax+b, 0\leq x \leq 1, \\ 0, □□   \end{cases} $$

?$E(x) = \frac{7}{12}$, ?a?b??????????

?: ???????????

??$E(X) = \int_{0}^{1}x(ax+b)dx = (\frac{1}{3}ax^3+\frac{1}{2}bx^2)\big|^1_0 = \frac{a}{3} + \frac{b}{2} = \frac{7}{12}$

$\int_{-\infty}^{+\infty}(ax+b)dx = \int_{0}^{1}(ax+b)dx = (\frac{1}{2}ax^2+bx)\big|^{1}_{0} = \frac{a}{2} + b = 1$

?? a=1, b=1/2,

???$0 \leq x \leq 1 $??

$F(x) = \int_{0}^{x}f(t)dt = \int_{0}^{x}(x+\frac{1}{2})dt = (\frac{x^2 + x}{2})\big|^x_0 = \frac{x^2 + x}{2} $

?????????

$$ F(x) = \begin{cases} 0, x \leq 0 \\ \big(\frac{x^2 + x}{2}), 0 \leq x \leq 1 \\ 1, x \gt 1 \end{cases} $$

4.1.2 ???????????

?X????????$g(x)$????????$Y=g(x)$???????????????????X ?????Y?????????????$g(x)$?????$E[g(x)]$, ?????????????????????

??4.1.1 ?X????????$Y=g(x)$,?$E(Y)$???

(1). ?X??????????????**

$$P{X=x_i} = p_i, i=1,2,3,....$$

?Y??????$$E(Y)=E[g(x)]=\Sigma_{i}^{+\infty}g(x_i)p_i$$

(2.)?X?????????????????$f(x)$? ?$\int_{-\infty}^{+\infty}g(x)f(x)dx$??????Y ??????

$$E(Y)=E[g(x)] = \int_{-\infty}^{+\infty}g(x)f(x)dx$$

? 4.1.6 ?????X????????

$x=x_i$	-2	0
P	0.3	0.1

?$E(2X+3)$, $E(X^2-1)$?

?? $E(2X+3) = 2 \times (-2) \times0.3 + 2\times 0 \times 0.1 = -1.2 $

$E(X^2-1)=4\times0.3 + 0^2 \times 0.1 = 1.2$

?? ?1???4.1.1????????$E[g(X)]$??????$g(X)$?????????X ??????????????????????????

??2???4.1.1 ?????????????.

4.1.3 ???????

?1??C?????$E(C) = C$

????????c???????????,$X \equiv C$???????

$$P{X=c} = 1 $$??$$E(c) = cP{X=c} = c $$

(2) ?C?????$E(CX) = CE(X)$

??? ?X???????????????$$P{X=x_i}=p_i, i=1,2,3,......$$

?$$E(CX) = \Sigma_{i}^{+\infty}(Cx_i)p_i = C\Sigma_{i}^{+\infty}x_ip_i = CE(x)$$

?X????????,??????$f(x)$, ?$$E(CX) = \int_{-\infty}^{+\infty}Cxf(x)dx= C\int_{-\infty}^{+\infty}xf(x)dx=CE(x)$$

(3)?????????????????????????????$$E(X+Y) = E(X) + E(Y)$$

(4) ?? ?????????????????????????$$E(\Sigma_{i}^{n}X_i) = \Sigma_{i}^{n}E(X_i)$$

(5) ?????????????????????????????????$$E(XY) = E(X)E(Y)$$

4.2 ???????

??

?????????????????????????????????????$$E{|X-E(x)|}$$

????????????????????? $$E{[X-E(X)]^2}$$?????????????????

??4.2.1 ?X????????$E[X-E(x)]^2$???????X????? $$D(X)=E[X-E(X)]^2$$

??

(1). ?????????X???$g(x)=[x-E(X)]^2$??;

(2). ?????X???????????$P{X=x_i}=p_i, i=1,2,3,.....$??$$D(X) = \Sigma_{i=1}^{+\infty}[X_i-E(X)]^2p_i $$

(3). ?X???????????????$f(x)$,?????$$D(X) = \int_{-\infty}^{+\infty}[x-E(X)]^2f(x)dx$$

4.2.2 ?????

?? 4.2.1 ??????X?????**???????????????????????????**?

???

$$ \begin{aligned} D(X) =E[X-E(X)]^2 \\ =E(X^2 - 2XE(X) + [E(X)]^2) \\ =E(X^2) - 2E(XE(x)) + [E(E(X))]^2 \\ =E(X^2) - 2E(X)E(X) + [E(X)]^2 \\ =E(X^2) - [E(X)]^2 \end{aligned} $$

4.2.3 ?????

(1). ???C,? $D(C)=0$

??$D(C)=E[C-E[C]]^2 = 0 ?? E(C) = C$

(2) ??? C?X ?????,? $D(CX) = C^2D(X)$

??

$$ \begin{align} D(X) =E[CX-E(CX)]^2 \\ =E(C^2X^2 - 2CXE(CX) + [E(CX)]^2) \\ =E(C^2X^2) - 2E(CXE(CX)) + [E(E(CX))]^2 \\ =E(C^2X^2) - 2E(X)E(CX)E(C) + [E(CX)]^2 \\ =C^2E(X^2) - 2C^2[E(X)]^2 + [CE(X)]^2 \\ =C^2[E(X^2) - [E(X)]^2] =C^2D(X) \end{align} $$

(3). ?$X?Y$?????????

$$D(X \pm Y) = D(X)+D(Y) \pm 2E{X-E(X)}E{Y-E(Y)}$$

????$X?Y????? $D(X+Y) = D(X) + D(Y)$

??

$$ \begin{align} D(X+Y) =E[X+Y - E(X+Y)]^2 \\ =E[(X+Y)^2 - 2(X+Y)E(X+Y) + [E(X+Y)]^2 \\ =D(X)+D(Y) + 2E{X-E(X)}E{Y-E(Y)} \end{align} $$

4.2.4 ????????????

? 4.2.1 ?????X??0-1????????$P{X=0} = 1-p, P{X=1} = p$? ?$E(X), D(X)$?

??$E(X) = \Sigma_{0}^{1} x_ip_i = 0 \times (1-p) + 1 \times p = p $

$D(X) = E(X^2) - [E(X)]^2 = 0^2 \times (1-p) + 1^2 \times p - p^2 = p - p^2$

? 4.2.2 ?$X \sim b(n,p)(????)? ?E(X), D(X)$

? ?? $X \sim b(n,p)$??X??n??????????$P{X=k}=C^k_pp^k(1-p)^{n-k}$?

$$ X = \begin{cases} 0 \text{ } , ?i??? \\ 1 \text{ } ??i??? \end{cases} i = 1,2,3,... \\ $$

$$ \begin{aligned} □ X=X_1+X_2+....+X_n \\ P{X=0} = 1-p0 \\ P{X=1} = p \\ E(X_i) = 0 \times (1-p) + p= p \\ E(X) = \Sigma_{i=1}^{n}E(Xi) = np \\ D(X) = np(1-p) \end{aligned} $$

?4.2.5 $X \sim P(\lambda)$??E(X),D(X)?

??????$P{X=k} = \frac{\lambda^k e^{-\lambda}}{k!}? k=0,1,2,3,.....$

$E(X) =\Sigma_{k=0}^{+\infty}k\frac{\lambda^k e^{-\lambda}}{k!} $

$ = \lambda e^{-\lambda} \Sigma_{k=0}^{+\infty}\frac{\lambda^{k-1}}{(k-1)!} = \lambda e^{-\lambda} e^{\lambda} = \lambda $

$E(X^2) = E[X(X-1)+X] = E[X(X-1)] + E(X) = \lambda^2 + \lambda $

$D(X) = E(X^2) - [E(X)]^2 = \lambda $

?4.2.6 ?$X \sim U(a,b) $?? $E(X),D(X)$?????

X ??????

$$ f(x) = \begin{cases} \frac{1}{b-a}□ \text{ } a \leq x \leq b \\ 0□ □□ \end{cases} $$

$E(X) = \frac{1}{b-a}\int_{a}^{b}xdx = \frac{a+b}{2}$

$D(X) = E(X^2)-[E(X)]^2 = \frac{1}{b-a}\int_{a}^{b}x^2dx = \frac{(b-a)^2}{12}$

?4.2.7 ? $X \sim E(\lambda),?? E(X),D(X)$?????

????????????

$$ f(x) = \begin{cases} \lambda e^{-\lambda x}□ \text{ } x \gt 0 \\ 0□ □□ \end{cases} $$

$E(X)= \int_{0}^{+\infty}x \lambda e^{ -\lambda x}dx = \frac{1}{\lambda}$

$E(X^2) = \frac{2}{\lambda^2}$

$D(X) = \frac{1}{\lambda^2}$

?4.2.8 ? $X \sim N(\mu, \sigma^2),?? E(X),D(X)$?????

????? $f(x) = \frac{1}{\sqrt{2 \pi \sigma}}e^{\frac{(x-\mu)^2}{2\sigma^2}}$

$E(X) = \mu, E(X^2) = \sigma^2 + \mu^2$

$D(X) = \sigma^2$

?? 4.2.2 ?????X??????$E(X)=\mu ,??D(X)= \sigma^2 $, ???????$\epsilon$, ???

$$P{|X- \mu | \geq \epsilon } \leq \frac{\sigma^2}{\epsilon^2} $$?

$$P{|X- \mu | \lt \epsilon } \geq 1- \frac{\sigma^2}{\epsilon^2} $$???????

4.1.3 ????????

??? ?????????????

?????? ?(X,Y)???????? $$E{[X-E(X)][Y-E(Y)]} $$??????????? (X,Y)???????$cov(X,Y)$?

?(X,Y)???????????????$$P{X=X_i, Y=y_i}=p_{ij}, i,j=1,2,3...,$$

? $$cov(X,Y) = \Sigma_{i,j}[x_i-E(X)](y_i-E(Y)]p_{i,j}$$

?(X,Y)?????????????????$f(x,y)$?????

?????$$cov(X,Y) =\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[x-E(X)](y-E(Y)]f(x,y)dxy $$

$$ \begin{aligned} cov(X,Y) = E{X-E(X)} \\ =E{XY - XE(Y) - YE(X) + E(X)E(Y)} \\ =E(XY) - E(X)E(Y)-E(Y)E(X) + E(X)E(Y) \\ =E(XY)-E(X)E(Y) \end{aligned} $$

?$X?Y ??$? $cov(X,Y) = 0$?

4.3.2 ??????

1. $cov(X,X) = D(X);$

2. $cov(X,Y) = cov(Y,X);$

3. $cov(aX,bY) = abcov(X,Y)$, ??ab?????

4. $cov(C,Y) = 0$,C?????

5. $cov(X_1+X_2, Y) = cov(X_1,Y) + cov(X_2,Y)$

????????????? $$D(X \pm Y) = D(X) + D(Y) \pm 2cov(X,Y) $$

?4.3.2 ?????(X,Y)??????

$$ f(x)=\begin{cases} 8xy, \text{ } 0 \leq x \leq y \leq 1 \\ 0, \text{ } □□ \end{cases} $$

?$cov(X,Y), D(X+Y)$

??$cov(X,Y) = E(XY)- E(X)E(Y)$

$f_X(x) = \int_{x}^{1}f_X(x,y)dy = 8x\int_{x}^{1}ydy = 4xy^2\big|^1_x = 4x(-x^2+1)$

$f_Y(y) = \int_{y}^{0}f_Y(x,y)dx = 8y\int_{0}^{y}xdx = 4yx^2\big|^y_0 = 4y^3$

$$ f_X(x)=\begin{cases} 4x(-x^2+1), \text{ } 0 \leq x \leq 1 \\ 0, \text{ } [illegible] \end{cases}
f_Y(x)=\begin{cases} 4y^3, \text{ } 0 \leq y \leq 1 \\ 0, \text{ } [illegible] \end{cases} $$

$E(X) = \int_{-\infty}^{+\infty}xf_X(x)dx = 8/15$

$E(Y) = \int_{-\infty}^{+\infty}yf_Y(x)dx = 4/5$

$E(XY) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}xyf(x,y)dxdy$

$= \int_{0}^{1} dx \int_{x}^{1}(xy \times 8xy)dy = 4/9 $

[illegible] ***D={(x,y)|0≤x≤y≤1}*** [illegible] [illegible] ***x=0***[illegible] ***y=1*** [illegible] ***y=x*** [illegible] ***(0,0)***[illegible] ***(0,1)***[illegible] ***(1,1)***[illegible] [illegible] ***f(x,y)*** [illegible] ***“***[illegible]***”*** [illegible]

[illegible]

?? x ??????? y ?????

[illegible] ***y*** [illegible] ***0*** [illegible] ***1*** [illegible] ***x*** [illegible] ***x*** [illegible] ***0*** [illegible] ***y***[illegible] ***x=0***[illegible] ***y=x*** [illegible] [illegible] ***y*** [illegible] ***x*** [illegible] ***0*** [illegible] ***y*** [illegible] ***“***[illegible]***”*** [illegible] [illegible] ***y*** [illegible] ***0*** [illegible] ***1*** [illegible] ***“***[illegible]***”*** [illegible]

?? y ??????? x ?????

[illegible] ***x*** [illegible] ***0*** [illegible] ***1*** [illegible] ***y*** [illegible] ***y*** [illegible] ***x*** [illegible] ***1***[illegible] ***y=x***[illegible] ***y=1***[illegible]

[illegible] ***x*** [illegible] ***y*** [illegible] ***x*** [illegible] ***1*** [illegible] ***“***[illegible]***”*** [illegible] [illegible] ***x*** [illegible] ***0*** [illegible] ***1*** [illegible] ***“***[illegible]***”*** [illegible]

$cov(X,Y) = E(XY) - E(X)(Y) = \frac{4}{9} - \frac{8}{15} \times \frac{4}{5} = \frac{4}{225}$

$E(X^2) = \int_{-\infty}^{+\infty}x^2f_X(x)dx = 1/3$

$E(Y^2) = \int_{-\infty}^{+\infty}y^2f_Y(x)dy = 2/3$

$D(X+Y) = D(X) + D(Y) + 2cov(X,Y)$

$ = E(X^2) - [E(X)]^2 + E(Y^2) - [E(Y)]^2 + \frac{4}{225} = 1/9$

4.3.3 ***??????? ?(X,Y)????????$ D(X) \gt 0, D(Y) \gt 0 $, ? $$ \rho = \frac{cov(X,Y)}{\sqrt{D(X)D(Y)}} $$?????X?Y ??????????$\rho=0$?X?Y????***

4.3.4 ???????

1. $|\rho| \leq 1;$

2. ?X?Y??????$\rho=0;$

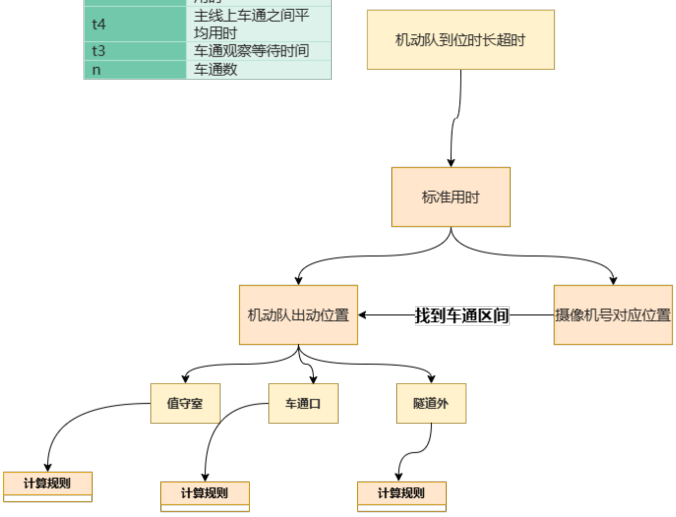
3. ?$D(X) >0, D(Y) > 0$?

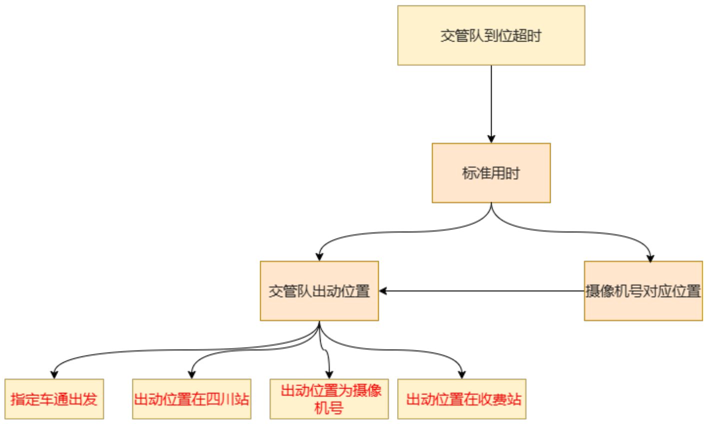
? $|\rho| =1$????????$a,b(a ??? 0)$??$P{Y=aX+b}=1$,

??? a>0 ? $\rho=1$,a<0? $\rho = -1$.

???

字母名称	说明
t	机动队到位标准用时
t1	准备时间 单位秒
t2	洞内车通之间平均用时
t4	主线上车通之间平均用时
t3	车通观察等待时间
n	车通数





比如左四车通
n=5-4
(1/2+1)t2
八车通
n= 8-6 =2
(1/2+2)t2

.值守室出发： (位置在5#-6#之间)

值守室到达最近车通时间(1/2+n)*t2